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Friday Nwabueze Ogana[1a+], Sacramento Corral-Rivas[2a], Jose Javier Gorgoso-Varela[3a]

# NONLINEAR MIXED-EFFECT HEIGHT-DIAMETER MODEL FOR *Pinus pinaster* AIT. AND *Pinus radiata* D. DON

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## HIGHLIGHTS

New generalized height-diameter model was developed for *Pinus pinaster* and *P. radiata*.

The properties and predictions of the models are biological reasonable.

The model compares well with other established height-diameter models used in quantitative forestry.

One tree selected at random was sufficient to calibrate the generalized model.

Keywords:
Hossfeld IV
Fixed-effect
Model calibration
Maritime pine
Monterrey pine

## ABSTRACT

Tree height-diameter (H-D) relationships are important for routine forest assessment. Several H-D relationships have been developed for different species and more are still evolving. This study introduces new H-D model developed for *Pinus pinaster* and *Pinus radiata* in Spain, based on data from 184 and 96 permanent sample plots, respectively, collected in the northwest region of the country. Nonlinear mixed-effect modelling technique was used to fit the generalized H-D model. The mixed-effect H-D model was calibrated using the random effects predicted from one to three randomly selected trees per sample plot. Different indices including root mean square error (RMSE) and adjusted coefficient of determination ($\bar{R}^2$) were used to assess the predictive performance of the model. The results showed that the new model had $\bar{R}^2$ and RMSE of 0.906 and 1.156 m and 0.814 and 1.703 m for *P. pinaster* and *P. radiata*, respectively. The calibration response involved the selection of one tree per sample plot and resulted in a reduction of RMSE by 6.5% and 13.5% for pinaster and P. radiata, respectively.

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+Correspondence:
ogana_fry@yahoo.com

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[1] University of Ibadan, Nigeria.- ORCID: 0000-0002-8388-204X[a]
[2] Sacramento Corral-Rivas: Instituto Tecnológico de El Salto, Durango, México - ORCID: 0000-0001-7624-0623[a]
[3] José Javier Gorgoso-Varela: föra forest technologies, Soria. Spain - ORCID: 0000-0002-8278-7876

## INTRODUCTION

Total height and diameter at breast height (D at 1.3 m above the ground) are fundamental tree variables that are routinely measured in forest inventory. They are required for the assessment of non-spatial structure of forest stands and estimation of volume (Adame et al., 2008; Gómez-García et al., 2014), basal area and determination of the competitive position of a tree in forest stand (West, 2015; Ogana, 2019a) and assessment of site productivity (Jayaraman and Lappi, 2001; West, 2015). Measurement of tree diameter at breast height is relatively simple, accurate and with low cost (Ferraz-Filho et al., 2018; Corral-Rivas et al., 2019). Conversely, tree height measurement is difficult, time consuming and expensive (Mehtätalo et al., 2015; Ozcelik et al., 2018). Owing to the associated problem with tree height measurement, only sub-sample of trees is measured. Thus, height-diameter (H-D) models are often used to estimate the height of trees for which the diameters have been measured (Kalbi et al., 2017).

Several nonlinear H-D models with single-variable (D) have been developed for different species (e.g. Yuancai and Parresol, 2001; Calama and Montero, 2004; Sharma, 2009; Ferraz-Filho et al., 2018). These models have been developed using either fixed-effect or mixed-effect techniques. In fixed-effect H-D models, the assumption of independence is violated (Ozcelik et al., 2018) and sufficient number of measurements is required for unbiased estimate of tree height (Arcangeli et al., 2014; Kalbi et al., 2017). On the other hand, mixed-effect models "allow the prediction of a response when using only the fixed-effect, and a calibrated response where random effects are predicted and included in the model using measurements of height from a sample trees" (Burkhart and Tome, 2012). Single-variable mixed-effect H-D models have been consistently used in the recent times (e.g. Sharma and Parton, 2007; Budhathoki et al., 2008; Zhang et al., 2016; Kalbi et al., 2017; Ogana, 2019a).

Corral-Rivas et al. (2019), asserted that modelling H-D relationships with single-variable (one predictor) [D] may not be adaptable to different stand dynamics and silvicultural conditions; and as such, may not possibly estimate all H-D relationships in the stands. This has led to the introduction of generalized H-D model so that different stand variations and conditions can be accounted for (Krisnawati et al., 2010). However, developing generalized H-D models often requires additional inventory costs, especially, model with mean height as one of the predictor variables (López-Sánchez et al., 2003). Some of the stand variables that have been used to develop generalized H-D models include number of trees per ha, basal area per ha, quadratic mean diameter, dominant and mean height, dominant diameter and age, among others. López-Sánchez et al. (2003) compared twenty-six fixed-effect generalized H-D models for *P. radiata* D. Don in Galicia, Spain using different stand variables. Recently, Corral-Rivas et al. (2019) compared ten generalized H-D models for seven pine species in Durango, Mexico.

Monterrey pine (*Pinus radiata*) and Maritime pine (*Pinus pinaster*) stands are important natural resources in northwest Spain. These species are considered as fast growing mainly occur in pure stands but sometimes *Pinus pinaster* also occurs in mixed stands. *Pinus* spp. and *Eucalyptus* spp. are the most commonly used species in productive stands in this area of Spain where the timber harvest represent more than 50% of the total country (Gorgoso-Varela et al., 2015). Pure stands of maritime pine are mainly derived from natural regeneration, although they are occasionally established as plantations. Exotic Monterrey pine stands are derived from plantations.

Therefore, the main objective of this study was to fit a new H-D function based on the variation of the Hossfeld IV model using fixed and mixed-effect models, and to compare it with other classical models. The *Pinus pinaster* and *Pinus radiata* stands in northwest Spain were used as case study.

## MATHERIAL AND METHODS

### Data

The data used for this study were obtained from two species of pine – Maritime pine (*Pinus pinaster* Ait) and Monterrey pine (*Pinus radiata* D. Don) in northwest Spain. The plantations and natural regeneration stands of *P. pinaster* cover 217,281 ha and 22,523 ha in the regions of Galicia and Asturias, respectively. The pure plantations stand of *P. radiata* occupy 96,177 ha and 25,385 ha in Galicia and Asturias, respectively (MMAMRM, 2011). The map of the study area is presented in Fig 1. A total of 184 permanent sample plots (PSPs) from *P. pinaster* stands and 96 PSPs from *P. radiata* stands were used for this study. The plot sizes ranged from 375 to 900 $m^2$; to achieve a minimum of 30 trees per plot. Square plots were used in this study. Diameter at breast height (D at 1.3 m above the ground) and total tree height (H) were measured with calliper and hypsometer to an accuracy of 0.1 cm and 0.1 m, respectively. A total of 17,845 and 12,722 trees were measured from *P. pinaster* and *P.*

*radiata*, respectively. The data from the two species were randomly split into two groups: 60% (fitting data) and 40% (validation data). The descriptive statistics of the quadratic mean diameter (dg, cm), basal area per ha (G, $m^2$ $ha^{-1}$), number of trees per ha (N trees $ha^{-1}$), dominant height ($H_o$, m), dominant diameter (i.e., average diameter of the 100 largest trees per ha; $D_o$, cm), mean height ($H_m$, m) and age (t, years) for the two species are presented in Table 1. The scatterplots showing the relationship between tree height and diameter for the species are shown in Fig. 2a and b.

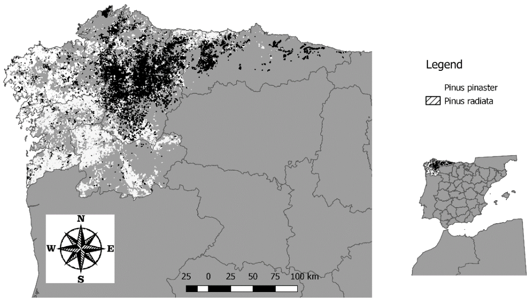


**FIGURE 1** Distribution of *Pinus pinaster* and *Pinus radiata* stands in northwest Spain (regions of Galicia and Asturias).

## Single variable Height-Diameter models

We derived a new H-D model from the Hossfeld IV growth function (Zeide 1993). The generic Hossfeld IV function is given by [1], where *H* = total tree height (m), *D* = diameter at breast height (1.3 m above the ground, cm), and *a*, *b*, *c* = model parameters. Preliminary application of the above function for height prediction *in P. pinaster* and *P. radiata* performed poorly – extremely large estimates for the parameters and larger standard errors, especially for *a* parameter. The inclusion of the point of measurement did not improve the function.

$$H = \frac{D^c}{(b + D^c)/_a} \qquad [1]$$

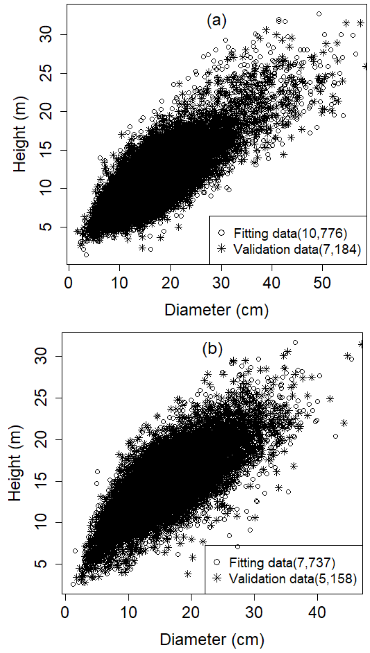


**FIGURE 2** Scatterplots of the relationship between tree height and diameter for (a) *Pinus pinaster* and (b) *Pinus radiata*.

To improve the Hossfeld IV function, the third parameter in the denominator was replaced with power 3. The rationale for this was because only the adjustment of parameter c achieved convergence and raised to power 3 since it was the third parameter that was replaced. The point of measurement of D was included. We termed this new growth function as modified Hossfeld IV model and expressed as [2]:

$$H = 1.3 + \frac{D^c}{(b + D^3)/_a} \qquad [2]$$

**TABLE 1** Descriptive statistics of the data groups.

| Species | Variables | Fitting data (N trees = 10776) | | | | Validation (N trees = 7184) | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Mean | SD | Min | Max | Mean | SD | Min | Max |
| P. pinaster (184 plots) | D (cm) | 17.4 | 7.0 | 1.8 | 56.7 | 17.4 | 6.8 | 1.6 | 63.2 |
| | H (m) | 11.5 | 3.8 | 1.4 | 32.7 | 11.4 | 3.7 | 2.3 | 36.0 |
| | G ($m^2$·$ha^{-1}$) | 34.4 | 12.2 | 7.8 | 115.1 | 34.3 | 12.0 | 7.8 | 115.1 |
| | dg (cm) | 18.0 | 5.4 | 10.4 | 41.5 | 17.9 | 5.2 | 10.4 | 41.5 |
| | Hm (m) | 11.4 | 3.4 | 4.7 | 24.3 | 11.3 | 3.3 | 4.7 | 24.3 |
| | Ho (m) | 13.1 | 3.7 | 5.5 | 30.6 | 13.0 | 3.6 | 5.5 | 30.6 |
| | t (years) | 19.8 | 8.1 | 8.0 | 61.5 | 19.6 | 7.9 | 8.0 | 61.5 |
| | N (trees·$ha^{-1}$) | 1466 | 510 | 375 | 2480 | 1465 | 507 | 375 | 2480 |
| | Do (m) | 26.1 | 6.9 | 13.8 | 55.8 | 26.0 | 6.6 | 13.8 | 55.8 |
| | | Fitting data (N trees = 7737) | | | | Validation (N trees = 5158) | | | |
| P. radiata (96 plots) | D (cm) | 16.3 | 6.1 | 1.3 | 45.5 | 16.4 | 6.2 | 1.8 | 47.1 |
| | H (m) | 14.6 | 3.9 | 2.6 | 31.7 | 14.7 | 4.0 | 2.8 | 31.5 |
| | G ($m^2$·$ha^{-1}$) | 30.7 | 7.1 | 14.3 | 46.6 | 30.7 | 7.1 | 14.3 | 46.6 |
| | dg (cm) | 17.1 | 2.7 | 11.9 | 27.1 | 17.2 | 2.8 | 11.9 | 27.1 |
| | Hm (m) | 14.5 | 2.4 | 9.7 | 22.6 | 14.5 | 2.4 | 9.7 | 22.6 |
| | Ho (m) | 17.8 | 3.0 | 12.9 | 28.7 | 17.9 | 3.1 | 12.9 | 28.7 |
| | t (years) | 17.1 | 3.8 | 12.0 | 32.0 | 17.1 | 3.6 | 12.0 | 32.0 |
| | N (trees $ha^{-1}$) | 1373 | 397 | 530 | 2610 | 1363 | 399 | 530 | 2610 |
| | Do (m) | 26.4 | 3.9 | 19.3 | 38.3 | 26.6 | 3.9 | 19.3 | 38.3 |

Max = maximum, min = minimum, SD = standard deviation, N trees = number of trees measured

The variables and the parameters in the model are previously defined in equation (1). The new model was compared with fifteen single-variable H-D models, where eight have 2-parameters and seven have 3-parameters. These models include: Bertalanffy, Curtis, Meyer, Michailoff, Michaelis-Menten (MM), Naslund, Power, Wykoff, Chapman-Richards (Richards), Gompertz, Korf, Logistic, Prodan, Ratkowsky and Weibull models (Table 2). These models have been consistently used in forestry including recent work by Corral-Rivas et al. (2019) and Ogana (2019a). The models were first fitted to the height-diameter data (fitting and validation) of *P. pinaster* and *P. radiata* using ordinary non-linear least square (ONLS) method, implemented in the '*nls*' function in R (R Core Team, 2017). Here all parameters in the models were considered as fixed.

## Generalized H-D models

The new function, that is, equation (2) was generalized to account for the different stand conditions. At first, different combinations of stand variables (as shown in Table 1 above) were evaluated. And it was observed that the inclusion of the quadratic mean diameter (*dg*), dominant height ($H_o$) and number of trees per ha (*N*) improved tree height prediction than other alternatives. The new model was termed generalized modified Hossfeld H-D model. It is expressed as [18], where *H* = total tree height, *D* = diameter at breast height, *dg* = quadratic mean diameter, $H_o$ = dominant height, *N* = number of trees per ha, *ln* = natural logarithm and *a*, *b*, *c*, *d*, e = model parameters.

$$H = 1.3 + \frac{D^c}{exp^{\left(\frac{-e}{ln(H_o)}\right)} \,{(b+D^3)}/{a}} \, exp^{\left({-d}/{\sqrt{{N}/{d_g}}}\right)} \qquad [18]$$

**TABLE 2** Single H-D models.

| Model name | Form | Reference | Eq. |
|---|---|---|---|
| Bertalanffy | $H = 1.3 + a\left(1 - e^{(-bD)}\right)^3$ | Von Bertalanffy (1957) | (3) |
| Curtis | $H = 1.3 + a(D/(1 + D))^b$ | Curtis (1967) | (4) |
| Meyer | $H = 1.3 + a(1 - e^{-bD})$ | Meyer (1940) | (5) |
| Michailoff | $H = 1.3 + ae^{(-bD^{-1})}$ | MIchailoff (1943) | (6) |
| Michaelis-Menten (MM) | $H = 1.3 + \frac{aD}{(b + D)}$ | Michaeli and Menten (1913) | (7) |
| Naslund | $H = 1.3 + \frac{D^2}{(a + bD)^2}$ | Näslund (1936) | (8) |
| Power | $H = 1.3 + aD^b$ | Stoffels and van Soest (1953) | (9) |
| Wykoff | $H = 1.3 + e^{a+b/(D+1)}$ | Wykoff et al. (1982) | (10) |
| Chapman-Richards | $H = 1.3 + a(1 - e^{-bD})^c$ | Richards (1959) | (11) |
| Gompertz | $H = 1.3 + ae^{\left(-be^{(-cD)}\right)}$ | Gompertz (1825), Huang et al. (1992) | (12) |
| Korf | $H = 1.3 + a\left(-e^{-bD^{-c}}\right)$ | Lundqvist (1957) | (13) |
| Logistic | $H = 1.3 + \frac{a}{1 + be^{-cD}}$ | Pearl and Reed (1920) | (14) |
| Prodan | $H = 1.3 + \frac{D^2}{a + bD + cD^2}$ | Strand (1959) | (15) |
| Ratkowsky | $H = 1.3 + ae^{-b/(D+c)}$ | Ratkowsky (1990) | (16) |
| Weibull | $H = 1.3 + a\left(1 - e^{bD^c}\right)$ | Yang et al. (1978) | (17) |

a, b, c = model parameters; H = total tree height (m); D = diameter at breast height (cm); e = base of the natural logarithm

The new generalized modified Hossfeld model was compared with eleven established generalized H-D models. Ten of the models have been used for *P. radiata* in Spain (López-Sánchez et al., 2003). While the generalized Michaelis-Menten (Gen.MM) was developed by Ogana (2019b). Different categories of generalized models were used including those ranging from 1 to 5 parameters. The parameters of the generalized models were considered as fixed. The models are presented in Table 3. The models were fitted to the fitting data sets from *P. pinaster* and *P. radiata* using ONLS, implemented in the '*nls*' function in R (R Core Team, 2017). The models were also validated.

## Model Evaluation and Assessment

Model assessment was based on residual graphs and numerical comparisons of the root mean square error (RMSE), adjusted coefficient of determination ($\bar{R}^2$), Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). The BIC evaluates a model based on its simplicity (Corral-Rivas et al., 2019). Models with low RMSE, AIC, and BIC and high $\bar{R}^2$ were regarded as good models. Based on these indices, the H-D models were ranked (Tewari and Singh, 2018). A value of 1 was assigned to the best model and the largest value was assigned to the worst model with respect to each fit index. The ranks were summed for each model; this was used as the indicator of the performance of the individual model with respect to the four fit indices. The smaller the rank sum, the better the model. The same indices for the H-D models were computed for the fitting and validation data sets from *P. pinaster* and *P. radiata*, Where: RSS = residual sum of square, n = sample size, p = number of parameters; $\bar{Y}_i$ = average total tree height; $Y_i$ is the observed tree height and $\hat{Y}_i$ is the theoretical value predicted by the model.

## Mixed-effects model

The new generalized modified Hossfeld model was refitted using mixed-effect models. In this technique both the within and between plot-height variability was considered by the introduction of random parameters. This technique also helps to overcome the issue of lack of independence between observations (Corral-Rivas et

**TABLE 3** Generalized H-D models.

| Model name | Form | Reference | Eq. |
|---|---|---|---|
| Canadas IV | $H = 1.3 + \left[b_0\left(\frac{1}{D} - \frac{1}{D_o}\right) + \left(\frac{1}{H_o - 1.3}\right)^{1/2}\right]^{-2}$ | Cañadas et al. (1999) | (19) |
| Mønness | $H = 1.3 + \left[b_0\left(\frac{1}{D} - \frac{1}{D_o}\right) + \left(\frac{1}{H_o - 1.3}\right)^{1/3}\right]^{-3}$ | Mønness (1982) | (20) |
| Gaffrey | $H = 1.3 + (H_o - 1.3)e^{b_0\left(1 - \frac{d_g}{D}\right) + b_1\left(\frac{1}{d_g} - \frac{1}{D}\right)}$ | Gaffrey (1988) | (21) |
| Sloboda | $H = 1.3 + a\left(1 - e^{(-bD)}\right)^3$ | Sloboda et al. (1993) | (22) |
| Gen.MM | $H = 1.3 + \frac{b_0 D}{(b_1 + D)} e^{\frac{-b_2}{\sqrt{H_o}}}$ | Ogana (2019b) | (23) |
| Pienaar | $H = b_0 H_o \left(1 - e^{\frac{-b_1 D}{d_g}}\right)^{b_2}$ | Pienaar (1991) | (24) |
| Mirkovich | $H = 1.3 + (b_0 + b_1 H_o - b_2 d_g)e^{-b_3/D}$ | Mirkovich (1958) | (25) |
| S.A I | $H = 1.3 + (b_0 + b_1 H_o - b_2 d_g)e^{-b_3/\sqrt{D}}$ | Schroder, Álvarez-González (2001) | (26) |
| Tome | $H = H_o e^{\left(b_0 + b_1 H_0 + b_2 \frac{N}{1000} + b_3 t\right)\left(\frac{1}{D} - \frac{1}{D_0}\right)}$ | Tomé (1989) | (27) |
| Cox III | $H = H_m \left[b_0 + b_1 H_m + b_2 \frac{H_m}{d_g} + b_3 D + b_4 \frac{N}{\frac{d_g(H_m d_g)}{D}}\right]$ | Cox (1994); López-Sánchez et al. (2003) | (28) |
| S.A II | $H = 1.3 + (b_0 + b_1 H_o - b_2 d_g + b_3 G)e^{-b_4/\sqrt{D}}$ | Schröder, Álvarez-González (2001) | (29) |

$b_0$, $b_1$, $b_2$, $b_3$, $b_4$ = model parameters; H = total tree height (m); D = diameter at breast height (cm); H0 = dominant height (m); D0 = dominant diameter (cm); dg = quadratic mean diameter (cm); t = age (years); G = basal area per ha (m2 ha-1); N = number of trees per ha (N trees ha-1); Hm =mean height (m); e = base of the natural logarithm;

al., 2019). The fixed and random parameters of a mixed-effects model are estimated simultaneously (Pinheiro and Bates, 1998). Following the methodology of Pinheiro and Bates (1998) and recently used by Mehtätalo et al. (2015) and Corral-Rivas et al. (2019), the non-linear mixed-effect was defined as [34], where $h_{ij}$ is the total tree height of tree *j* on plot *i* and corresponding diameter $d_{ij}$; *f* is the nonlinear model; $\boldsymbol{\phi}_i$ is the parameter vector *r* x *1* where *r* represents the whole parameters in the model. The lambda λ is a *p* x *1* vector for the fixed parameters (*a*, *b* and *c*) and *p* is the number of parameters. The $b_i$ represent the *q* x *1* vector for the random parameters (*q* = number of random parameters) – it is the plot effect that shows the variation in the parameters of plot *i* from the typical plot. $A_i$=r x p and $B_i$=r x q, these are the dimensional matrix for both the fixed and random effects, respectively, unique to plot *i* (Corral-Rivas et al., 2019). The underlying assumption is that the "plot effects have a common multivariate normal distribution with mean 0 and variance-covariance matrix var($b_i$)=D for all values of *i*" (Mehtätalo et al., 2015). The epsilon is the vector form of the random error and is assumed to be normal and independent with zero mean and constant variance var($\varepsilon_{ij}$) = $\sigma^2$. To decide which fixed parameter should be considered as random, different combinations of fixed and random parameters were evaluated using RMSE, $\bar{R}^2$ , AIC, and BIC.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{n}} \quad [30]$$

$$\bar{R}^2 = 1 - \frac{(n-1)\sum_{i=1}^{n}(Y_i - Y_i)^2}{(n-p)\sum_{i=1}^{n}(Y_i - \bar{Y}_i)^2} \quad [31]$$

$$AIC = n \ln\left(\frac{RSS}{n}\right) + 2p \quad [32]$$

$$BIC = n \ln\left(\frac{RSS}{n}\right) + p \ln n \quad [33]$$

$$h_{ij} = f(d_{ij}; \boldsymbol{\phi}_i) + \varepsilon_{ij} \quad [34]$$

$$\boldsymbol{\phi}_i = \boldsymbol{A}_i \boldsymbol{\lambda} + \boldsymbol{B}_i \boldsymbol{b}_i \quad [35]$$

The parameters of the mixed-effects models were estimated with the method of restricted maximum likelihood (REML) to diminish biases through the '*nlme*' function in R (R Core Team, 2017). The empirical best linear unbiased predictor (EBLUP) approximation was used for the maximization of the marginal likelihood function as recommended by Beal and Sheiner (1982). The whole data sets were used for the mixed model.

## Calibration of the mixed models

Predicting the response variable (total tree height) with the random effects estimated from its prior information is termed as calibration of the mixed-effects H-D model (Sharma et al. 2019). Prediction of the random effect for a given stand and the adjustment of the fixed part of the mixed-effects H-D model requires the measured height of one or more trees in each sample plot to be used to predict the specific random parameters for that stand. We calibrated the mixed-effects H-D model using the random effects predicted from one to three randomly selected trees per sample plot in the validation data set. The following procedures were used in selecting the sample trees:

One sample tree: the tree nearest to the 25th quantile;

Two sample trees: each one nearest to the 50th and 75th quantiles

Three sample trees: each one nearest to the quadratic mean diameter, minimum and maximum diameter

To make predictions for the random components using a sub-sample of heights the empirical best linear unbiased predictor (EBLUP) theory was applied following the expression 36 (Vonesh and Chinchilli, 1997), where $\hat{D}$ is a random effect vector that describes plot-level H-D variations for the $i^{th}$ plot; $\hat{R}_i$ is the $m_j \times m_j$ variance–covariance matrix for within-plot variability; $Z_i$ is $m \times q$ matrix of the partial derivatives of the valued random parameters from $\hat{b}_i$ and $\hat{\varepsilon}_i$ $m \times 1$ residuals vector, whose components result from the difference between the observed height of each tree and the predicted value from the model, considering only fixed parameters. The values were estimated iteratively, for that purpose, we developed an R script using matrix functions in R (R Core Team, 2017). The calibration alternatives were evaluated in terms of the previously defined statistics (RMSE and $\bar{R}^2$ ) and compared with the RMSE estimations obtained with ONLS in the individual fit of the selected model to each of the calibration plots.

$$\hat{b}_i = \hat{D}\hat{Z}_i^T\left(\hat{R}_i + \hat{Z}_i\hat{D}\hat{Z}_i^T\right)^{-1}\hat{\varepsilon}_i \qquad [36]$$

## RESULTS AND DISCUSSION

### Single variable fixed-effect models

In the fitting data set of *P. pinaster*, the RMSE, $\bar{R}^2$ , AIC and BIC of the models ranged from 2.291 – 2.649 m, 0.513 – 0.636, 48453 – 51589 and 48482 – 51611, respectively (Table 4). When the models were validated, the RMSE, $\bar{R}^2$ , AIC and BIC values of the models ranged from 2.249 – 2.591 m, 0.514 – 0.634, 11649 – 13685 and 11669 – 13698, respectively. The assessment of the models based on their rank positions with respect to the fit indices showed that modified Hossfeld function ranked 4th behind Gompertz, Ratkowsky and Logistic functions for the fitting data. It also ranked 6th for the validation data set. Michailoff and Bertalanffy had highest rank sum of 60 (15th) and 64 (16th), respectively.

In the case of *P. radiata*, the results from the fitting data set showed that the RMSE, $\bar{R}^2$ , AIC and BIC values of the models ranged from 2.452 – 2.704 m, 0.523 – 0.607, 35843 – 37350 and 35871 – 37371, respectively (Table 5). The fit indices of the models for the validation data were in the range of 2.513 – 2.788 m, 0.516 – 0.607, 9510 – 10582 and 9530 – 10596, respectively. Evaluation of the models based on their relative position showed that modified Hossfeld IV was the best performed model with the lowest ranks sum of 4 (1st) for both fitting and validation data sets. This was followed by Power and Ratkowsky with rank sum of 8 (3rd) and 12 (4th), respectively. Weibull and Bertalanffy had highest rank sum of 60 (15th) and 64 (16th), respectively. Other H-D models including Chapman-Richards, Curtis, Korf, Meyer, Michaelis-Menten (MM), Nalsund and Wykoff performed relatively. The fit indices provided identical ranks, especially the RMSE and $\bar{R}^2$ in both species.

To further assess the performance of the single-variable H-D models, the average residuals of the predicted height from the validation data set were computed for different diameter classes and plotted for the two species. A diameter class of 5 cm interval was used and the mean residual in height prediction was assessed. Only the residual graphs of the two best single-variable H-D models and the modified Hossfeld function for *P. pinaster* and *P. radiata* were presented (Fig 3a and b). The graphs showed that modified Hossfeld IV function had the same behavior as Gompertz and Logistic models. These models both overestimated and underestimated tree height in the lower (< 5 cm) and larger (> 55 cm) diameter classes, respectively in *P. pinaster* (Fig 3a). In the case of *P. radiata*, the mean residuals plot of the modified Hossfeld function was more stable than Ratkowsky and Power H-D models. The modified Hossfeld was consistent across the diameter classes except in the large classes (> 40 cm). In contrast, Ratkowsky and Power models both overestimated and underestimated tree height in the lower (< 5 cm) and larger (≥ 40 cm) diameter classes.

The performance of the new modified Hossfeld in *P. pinaster* and *P. radiata* was more stable than most of the functions evaluated in this study. The parameter estimates are significant with smaller standard errors. This is a new single variable H-D model. The Logistic and Gompertz functions with the least bias in *P. pinaster*, did not perform well in *P. radiata*. This implies that the nature of data could affect the performance of a function. Mehtätalo et al. (2015) selected the logistic function for modelling H-D relationships for the pure Scots pine. They reported a better performance with the logistic function compared to other models. Similar observation was reported in Ogana (2018) for *Gmelina arborea* Roxb stands. Another function which seems to be relatively stable is the Ratkowsky. Its ranked 2nd and 3rd in *P. pinaster* and *P. radiata*, respectively. This model was selected by Liu *et al*. (2018) as the most suitable non-linear model between 32 H-D models evaluated for Metasequoia in China.

When considering model simplicity and the ease of fitting the functions, the new modified Hossfeld HD model could be adopted for estimating tree height especially, in the *P. radiata* stand.

### Generalized h-d models

The results of the generalized H-D models are presented in Table 6 and 7. In *P. pinaster*, the new generalized Hossfeld model had RMSE, $\bar{R}^2$ , AIC and BIC of 1.290, 0.885, 36076 and 36120, respectively

**TABLE 4** Parameter estimates and fit indices for both fitting and validation data set of *Pinus pinaster*.

| Model | Parameter estimates | | | Fitting data | | | | | Validation data | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | a | b | c | RMSE | $\bar{R}^2$ | AIC | BIC | ∑Rank | RMSE | $\bar{R}^2$ | AIC | BIC | ∑Rank |
| Bertalanffy | 16.45(0.095) | 0.117(0.001) | | 2.649 | 0.513 | 51589 | 51611 | $64_{16}$ | 2.591 | 0.514 | 13685 | 13698 | $64_{16}$ |
| Curtis | 23.885(0.170) | 14.306(0.131) | | 2.504 | 0.565 | 50370 | 50392 | $56_{14}$ | 2.450 | 0.566 | 12878 | 12892 | $56_{14}$ |
| Meyer | 34.36(0.814) | 0.021(0.001) | | 2.335 | 0.622 | 48864 | 48886 | $32_{8}$ | 2.287 | 0.621 | 11892 | 11906 | $32_{8}$ |
| Michailoff | 23.262(0.163) | 13.408(0.125) | | 2.521 | 0.559 | 50517 | 50539 | $60_{15}$ | 2.466 | 0.56 | 12974 | 12988 | $60_{15}$ |
| MM | 56.677(1.526) | 78.169(2.667) | | 2.33 | 0.623 | 48822 | 48844 | $28_{7}$ | 2.283 | 0.623 | 11864 | 11878 | $28_{7}$ |
| Naslund | 2.229(0.019) | 0.18(0.001) | | 2.409 | 0.597 | 49542 | 49563 | $48_{12}$ | 2.359 | 0.598 | 12333 | 12347 | $48_{12}$ |
| Power | 1.145(0.019) | 0.768(0.005) | | 2.305 | 0.632 | 48580 | 48602 | $19_{5}$ | 2.259 | 0.631 | 11714 | 11728 | $16_{4}$ |
| Wykoff | 3.20(0.007) | -15.26(0.137) | | 2.487 | 0.571 | 50227 | 50248 | $52_{13}$ | 2.434 | 0.572 | 12784 | 12798 | $52_{13}$ |
| Chapman-Richards | 25.00(0.666) | 0.04(0.002) | 1.30(0.035) | 2.396 | 0.602 | 49420 | 49449 | $44_{11}$ | 2.343 | 0.603 | 12240 | 12261 | $44_{11}$ |
| Gompertz | 36.087(1.306) | 2.322(0.023) | 0.03(0.001) | 2.291 | 0.636 | 48453 | 48482 | $4_{1}$ | 2.249 | 0.634 | 11649 | 11669 | $4_{1}$ |
| Korf | 70.00(9.374) | 6.00(0.088) | 0.40(0.029) | 2.375 | 0.609 | 49228 | 49257 | $40_{10}$ | 2.322 | 0.610 | 12110 | 12131 | $40_{10}$ |
| Logistic | 27.647(0.548) | 5.869(0.094) | 0.069(0.001) | 2.291 | 0.636 | 48459 | 48489 | $7_{3}$ | 2.250 | 0.634 | 11658 | 11679 | $9_{3}$ |
| Prodan | -3.455(0.074) | 1.746(0.014) | 0.009(0.001) | 2.305 | 0.632 | 48587 | 48615 | $21_{6}$ | 2.259 | 0.631 | 11716 | 11736 | $18_{5}$ |
| Ratkowsky | 96.23(6.848) | 116.69(8.033) | 34.39(1.923) | 2.291 | 0.636 | 48459 | 48488 | $6_{2}$ | 2.249 | 0.634 | 11653 | 11673 | $6_{2}$ |
| Weibull | 85.00(23.107) | 0.01(0.002) | 0.900(0.023) | 2.337 | 0.621 | 48885 | 48914 | $36_{9}$ | 2.295 | 0.619 | 11940 | 11960 | $36_{9}$ |
| *Hossfeld | 1.134(0.019) | -3.555(0.773) | 3.771(0.006) | 2.304 | 0.632 | 48573 | 48602 | $16_{4}$ | 2.261 | 0.630 | 11727 | 11748 | $24_{6}$ |

*= modified Hossfeld IV; a, b, c = model parameters; values in parenthesis are the standard errors; subscripts represent relative positions of the rank sum

**TABLE 5** Parameter estimates and fit indices for both fitting and validation data set of *Pinus radiata*.

| Model | Parameter estimates | | | Fitting data | | | | | Validation data | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | a | b | c | RMSE | $R^2$ | AIC | BIC | ∑Rank | RMSE | $R^2$ | AIC | BIC | ∑Rank |
| Bertalanffy | 17.105(0.070) | 0.178(0.001) | | 2.704 | 0.523 | 37350 | 37371 | $64_{16}$ | 2.788 | 0.516 | 10582 | 10596 | $64_{16}$ |
| Curtis | 24.001(0.149) | 8.96(0.102) | | 2.537 | 0.58 | 36368 | 36388 | $52_{13}$ | 2.623 | 0.575 | 9914 | 9927 | $53_{13}$ |
| Meyer | 22.748(0.244) | 0.058(0.001) | | 2.484 | 0.597 | 36042 | 36063 | $36_{9}$ | 2.552 | 0.595 | 9668 | 9681 | $36_{9}$ |
| Michailoff | 23.434(0.142) | 8.272(0.095) | | 2.55 | 0.575 | 36447 | 36468 | $56_{14}$ | 2.627 | 0.570 | 9969 | 9982 | $56_{14}$ |
| MM | 33.362(0.481) | 23.117(0.587) | | 2.47 | 0.602 | 35952 | 35973 | 195 | 2.534 | 0.600 | 9603 | 9616 | $26_{7}$ |
| Naslund | 1.218(0.014) | 0.193(0.001) | | 2.500 | 0.592 | 36141 | 36162 | $44_{11}$ | 2.572 | 0.588 | 9748 | 9761 | $44_{11}$ |
| Power | 2.642(0.044) | 0.587(0.006) | | 2.455 | 0.606 | 35860 | 35880 | $8_{2}$ | 2.515 | 0.606 | 9518 | 9531 | $8_{2}$ |
| Wykoff | 3.203(0.006) | -9.696(0.108) | | 2.525 | 0.584 | 36294 | 36315 | $48_{12}$ | 2.600 | 0.579 | 9863 | 9876 | $48_{12}$ |
| Chapman-Richards | 25.322(0.156) | 0.040(0.007) | 0.832(0.008) | 2.468 | 0.602 | 35939 | 35960 | $16_{4}$ | 2.534 | 0.600 | 9597 | 9610 | $24_{6}$ |
| Gompertz | 24.911(0.573) | 1.699(0.017) | 0.063(0.003) | 2.470 | 0.602 | 35953 | 35981 | $21_{6}$ | 2.531 | 0.601 | 9585 | 9605 | $21_{5}$ |
| Korf | 35.00(2.196) | 5.00(0.195) | 0.60(0.037) | 2.496 | 0.593 | 36117 | 36145 | $40_{10}$ | 2.566 | 0.590 | 9727 | 9746 | $40_{10}$ |
| Logistic | 22.808(0.378) | 3.318(0.056) | 0.097(0.003) | 2.478 | 0.599 | 36009 | 36037 | $32_{8}$ | 2.539 | 0.599 | 9620 | 9639 | $32_{8}$ |
| Prodan | -1.243(0.019) | 0.875(0.010) | 0.024(0.001) | 2.473 | 0.601 | 35974 | 36002 | $28_{7}$ | 2.527 | 0.603 | 9567 | 9587 | $15_{4}$ |
| Ratkowsky | 39.943(1.547) | 31.196(2.221) | 12.71(0.996) | 2.462 | 0.604 | 35904 | 35931 | $12_{3}$ | 2.524 | 0.603 | 9556 | 9575 | $12_{3}$ |
| Weibull | 25.00(1.807) | 0.110(0.004) | 0.070(0.033) | 2.553 | 0.574 | 36465 | 36493 | $60_{15}$ | 2.632 | 0.569 | 9990 | 10010 | $60_{15}$ |
| *Hossfeld | 2.819(0.065) | 12.995(3.530) | 3.566(0.008) | 2.452 | 0.607 | 35843 | 35871 | $4_{1}$ | 2.513 | 0.607 | 9510 | 9530 | $4_{1}$ |

*= modified Hossfeld IV; values in parenthesis are the standard errors; subscripts represent relative positions of the rank sum

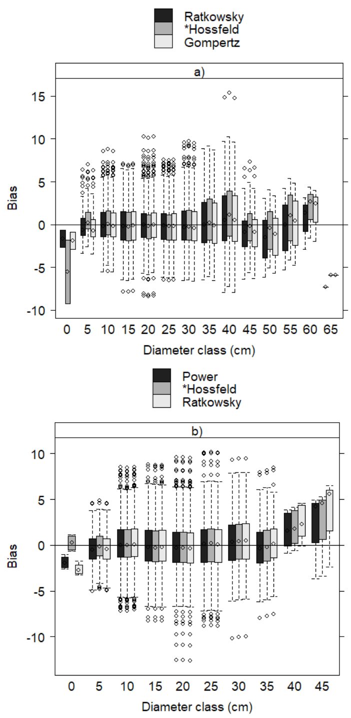


**FIGURE 3** Mean residual plot against diameter classes for the single-variable H-D model in (a) *P. pinaster* and (b) P. *radiata* (validation data).

for the fitting data set; and 1.271, 0.883, 3454 and 3489, respectively for the validation data. The model ranked 3rd behind Sloboda (1st) and Tomé (2nd) models both for the fitting and validation data sets. Mirkovich and Gaffrey had the highest rank sum of 44 (11th) and 48 (12th), respectively. In the case of *P. radiata*, the new generalized Hossfeld model had the lowest rank sum of 4 for both the fitting and validation data sets and as such, was the best H-D model. Its RMSE, $\bar{R}^2$, AIC and BIC were 1.775, 0.794, 30848 and 30890, respectively for the fitting data set; and 1.841, 0.789, 6307 and 6340, respectively for the validation data. This was followed by Schroder and Alvarez (S.A II) and S.A I. Sloboda and Gaffrey had the poorest results. The inclusion of stand variables (*dg*, $H_o$ and *N*) improved the prediction of the modified Hossfeld model. The RMSE decreased from 2.261 to 1.271 (44%) and from 2.513 to 1.841 (27%) in *P. pinaster* and *P. radiata*, respectively. This is expected because the introduction of stand variables in a model usually decrease the variability within sample unit. Several researchers have recommended the inclusion of stand variables in H-D relationships (e.g., López-Sánchez et al., 2003; Canga-Libano et al., 2009; Crecente-Campo et al.,

**TABLE 6** Parameter estimates and fit indices for the generalized H-D models of *P. pinaster* and *P. radiata*.

| Model | Parameter estimates | | | | | Fitting data | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $b_0$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | RMSE | $\bar{R}^2$ | AIC | BIC | ∑Rank |
| | | | | | *P. pinaster* | | | | | |
| Canadas | 0.992(0.008) | | | | | 1.292 | 0.884 | 36102 | 36117 | $16_4$ |
| Monness | 0.982(0.007) | | | | | 1.301 | 0.883 | 36255 | 36269 | $23_6$ |
| Gaffrey | 0.278(0.019) | 2.447(0.340) | | | | 1.797 | 0.776 | 43221 | 43243 | $48_{12}$ |
| Sloboda | -0.411(0.005) | 0.014(0.001) | | | | 1.271 | 0.888 | 35759 | 35781 | $4_1$ |
| Gen.MM | 95.899(0.818) | 9.976(0.177) | 6.352(0.042) | | | 1.309 | 0.881 | 36390 | 36419 | $36_9$ |
| Piennaar | 1.109(0.012) | 1.498(0.086) | 0.805(0.036) | | | 1.310 | 0.881 | 36407 | 36437 | $40_{10}$ |
| Mirkovich | 2.844(0.096) | 1.145(0.011) | 0.158(0.007) | 6.273(0.007) | | 1.313 | 0.881 | 36456 | 36492 | $44_{11}$ |
| S.A I | 5.665(0.179) | 1.741(0.021) | 0.302(0.012) | 3.346(0.037) | | 1.305 | 0.882 | 36328 | 36364 | $28_7$ |
| Tome | -2.508(0.254) | -0.277(0.163) | 0.770(0.083) | -0.026(0.008) | | 1.284 | 0.886 | 35976 | 36013 | $8_2$ |
| Cox III | 0.718(0.010) | -0.029(0.001) | 0.559(0.014) | 0.017(0.001) | -0.42(0.024) | 1.305 | 0.882 | 36336 | 36380 | $30_8$ |
| SA II | 5.654(0.178) | 1.663(0.022) | 0.311(0.012) | 0.035(0.004) | 3.349(0.037) | 1.299 | 0.883 | 36238 | 36282 | $20_5$ |
| *Hossfeld | 40.728(1.104) | 34.663(3.674) | 3.368(0.006) | 0.573(0.035) | -5.988(0.043) | 1.290 | 0.885 | 36076 | 36120 | $12_3$ |
| | | | | | *P. radiata* | | | | | |
| Canadas | 0.972(0.007) | | | | | 1.812 | 0.786 | 31158 | 31172 | $26_7$ |
| Monness | 1.005(0.007) | | | | | 1.817 | 0.784 | 31205 | 31219 | $34_9$ |
| Gaffrey | 0.424(0.037) | 0.579(0.638) | | | | 2.833 | 0.476 | 38078 | 38099 | $48_{12}$ |
| Sloboda | -0.486(0.006) | 0.028(0.002) | | | | 1.941 | 0.754 | 32227 | 32249 | $44_{11}$ |
| Gen.MM | 122.169(2.221) | 14.567(0.264) | 6.492(0.081) | | | 1.817 | 0.784 | 31203 | 31231 | $33_8$ |
| Piennaar | 1.121(0.012) | 1.466(0.067) | 0.964(0.033) | | | 1.801 | 0.788 | 31069 | 31097 | $16_4$ |
| Mirkovich | 6.502(0.241) | 1.101(0.015) | 0.255(0.016) | 7.200(0.068) | | 1.811 | 0.787 | 31159 | 31193 | $25_6$ |
| S.A I | 12.959(0.441) | 1.825(0.029) | 0.512(0.028) | 3.956(0.035) | | 1.787 | 0.791 | 30948 | 30983 | $12_3$ |
| Tome | -4.676(0.377) | -0.154(0.016) | 1.554(0.124) | -0.061(0.012) | | 1.807 | 0.787 | 31121 | 31156 | $20_5$ |
| Cox III | 0.699(0.016) | -0.042(0.001) | 0.659(0.017) | 0.025(0.001) | -2.032(0.092) | 1.827 | 0.782 | 31291 | 31333 | $40_{10}$ |
| S.A II | 12.175(0.447) | 1.803(0.028) | 0.565(0.029) | 0.065(0.009) | 3.949(0.035) | 1.780 | 0.793 | 30895 | 30937 | $8_2$ |
| *Hossfeld | 59.376(2.467) | 49.397(3.976) | 3.436(0.007) | 1.413(0.068) | -7.197(0.089) | 1.775 | 0.794 | 30848 | 30890 | $4_1$ |

*= modified Hossfeld IV; $b_0$, $b_1$, $b_2$, $b_3$, $b_4$ = estimated parameter; values in parenthesis are the standard errors; subscripts represent relative positions of the rank sum.

**TABLE 7** Fit indices of the generalized H-D models for validation data of *Pinus pinaster* and *Pinus radiata*.

| *Pinus pinaster* | | | | | | *Pinus radiata* | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Model | RMSE | $\bar{R}^2$ | AIC | BIC | ∑Rank | Model | RMSE | $\bar{R}^2$ | AIC | BIC | ∑Rank |
| *Canadas* | *1.277* | *0.882* | *3511* | *3518* | $19_5$ | *Canadas* | *1.881* | *0.780* | *6517* | *6524* | $28_7$ |
| Monness | 1.287 | 0.880 | 3625 | 3632 | $24_6$ | Monness | 1.885 | 0.779 | 6543 | 6550 | $40_{10}$ |
| Gaffrey | 1.780 | 0.771 | 8290 | 8304 | $48_{12}$ | Gaffrey | 2.893 | 0.479 | 10964 | 10977 | $48_{12}$ |
| Sloboda | 1.255 | 0.886 | 3271 | 3285 | $4_1$ | Sloboda | 1.982 | 0.755 | 7059 | 7072 | $44_{11}$ |
| Gen.MM | 1.293 | 0.879 | 3703 | 3724 | $29_7$ | Gen.MM | 1.877 | 0.781 | 6501 | 6521 | $21_6$ |
| Piennaar | 1.302 | 0.877 | 3795 | 3815 | $40_{10}$ | Piennaar | 1.881 | 0.780 | 6523 | 6543 | $32_9$ |
| Mirkovich | 1.312 | 0.876 | 3909 | 3937 | $44_{11}$ | Mirkovich | 1.881 | 0.780 | 6521 | 6541 | $30_8$ |
| S.A I | 1.299 | 0.878 | 3771 | 3799 | $36_9$ | S.A I | 1.853 | 0.786 | 6371 | 6397 | $13_3$ |
| Tome | 1.268 | 0.884 | 3416 | 3443 | $8_2$ | Tome | 1.879 | 0.789 | 6512 | 6538 | $19_5$ |
| Cox III | 1.275 | 0.882 | 3504 | 3539 | $17_4$ | Cox III | 1.870 | 0.782 | 6468 | 6501 | $17_4$ |
| S.A II | 1.293 | 0.879 | 3701 | 3735 | $29_7$ | S.A II | 1.846 | 0.788 | 6336 | 6369 | $9_2$ |
| *Hossfeld | 1.271 | 0.883 | 3454 | 3489 | $12_3$ | *Hossfeld | 1.841 | 0.789 | 6307 | 6340 | $4_1$ |

S.A = Schroder and Alvarez; Gen.MM = generalized Michaelis-Menten

2010; Uzoh, 2017). Corral-Rivas et al. (2019) compared ten generalized H-D models for seven pine species in Durango, Mexico. They asserted that modelling H-D relationship with D only may not be adaptable to different stand dynamics and silvicultural conditions; and as such, may not possibly estimate all H-D relationships in the stands.

Also, the mean residuals graph of the generalized Hossfeld showed similar behavior with Sloboda and Tome H-D models for *P. pinaster*, and SA I and SA II in *P. radiata* (Fig 4a and b). The models both overestimated and underestimated tree height in the lower (> 5 cm) and larger (> 55 cm) diameter classes, respectively in *P. pinaster* (Fig 4a).

## Generalized Hossfeld with Mixed-effect

The result from the adjustment of the new generalized modified Hossfeld model with mixed effects is presented in Table 8. Of the different combination of fixed and random parameter tried, the best fit was found by relating the parameter $b_4$ with a random parameter $u_j$ in an additive form ($b_4 + u_j$). The model is represented by equation (37). The estimated values and signs of all parameters are biologically plausible and interpretable. The predicted trajectories with new generalized height-diameter model showed appropriate trends, logical asymptotes, and adaptation at observed values; therefore, predictions out of the range of the

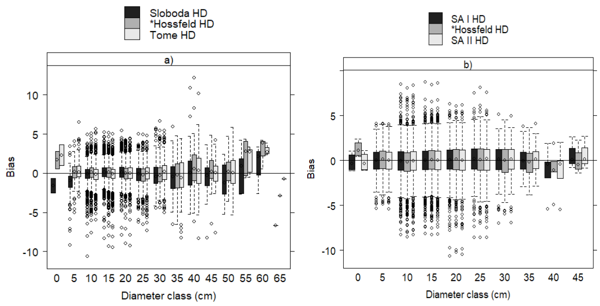


**FIGURE 4** Mean residual plot against diameter classes for the generalized H-D in (a) P. pinaster and (b) P. radiata (validation data).

used dataset could be done within reasonable limits, as the biological behaviour of the transition functions was adequate. The result showed that the inclusion of the random parameter improved the generalized Hossfeld function with RMSE and $\bar{R}^2$ of 1.156 and 0.906, respectively for *P. pinaster*; 1.703 and 0.814, respectively for *P. radiata*. Also, the graph of residual against predicted height showed homogeneous variance (homoscedastic) in both species (Fig. 5a and b), where $u_j \sim N\,[0, \sigma_u]$ is the random parameter which is assumed to be normal with a zero mean and constant variance due to random effect ($\sigma_u^2$). Other parameters in the model were previously defined in equation (18).

$$H_{ij} = 1.3 + \frac{D_{ij}^{\ b_2}}{exp^{\left(\frac{-b_4+u_j}{ln(H_o)}\right)} \left(b_1 + D_{ij}^{\ 3}\right) / b_0} exp^{\left( -b_3 / \sqrt{N/d_g} \right)} \quad [37]$$

**TABLE 8** Parameters estimated, variance components and fit indices of the mixed-effect H-D model (equation 37) for both two species.

| Parameters | *P. pinaster* | *P. radiata* |
|---|---|---|
| | Fixed parameters | |
| $b_0$ | 34.759 | 57.021 |
| $b_1$ | 41.344 | 46.516 |
| $b_2$ | 3.367 | 3.441 |
| $b_3$ | 0.358 | 1.211 |
| $b_4$ | -5.651 | -7.199 |
| | Variance components | |
| $\sigma^2 u$ | 0.022 | 0.013 |
| $\sigma^2$ | 1.439 | 2.92 |
| | Fit indices | |
| RMSE | 1.156 | 1.703 |
| $\bar{R}^2$ | 0.906 | 0.814 |
| AIC | 56171 | 50329 |
| BIC | 56218 | 50373 |

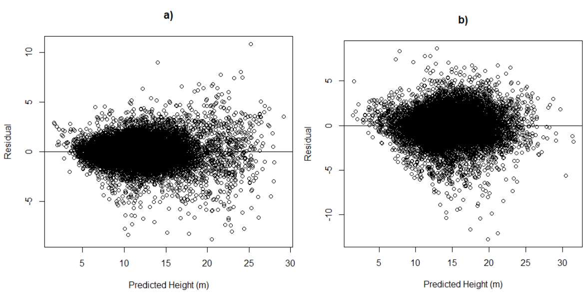


**FIGURE 5** Residual plot against predicted height for the Hossfeld generalized model with mixed effect in (a) *P. pinaster* and (b) *P. radiata*.

The mixed-effects H-D model developed (equation 37) was calibrated with the random effect predicted from measured heights of one to three randomly selected trees on each sample plot. The calibrated response described 89.9% and 70.6% of H-D variations in *P. pinaster* and *P. radiata*, respectively with one tree selected randomly (Table 9). It was observed that when one tree was selected randomly to calibrate the model, the RMSE value was reduced by 6.5% with respect to the estimated value from equation (18) by ONLS in *P. Pinaster*. On the other hand, in *P. radiata* the RMSE value was reduced by 13.5% with one randomly tree with respect to the ONLS fitting. The Hossfeld generalized H-D model with mixed-effect was precise enough for predicting the total height tree for the species studied. The importance of calibrating mixed-effect models to the forest owners cannot be overemphasized because only few sample trees are required to provide information on the height of all trees in the stand (Corral-Rivas *et al.*, 2019). Different sample trees for model calibration have been reported. For example, Castedo-Dorado et al. (2006) recommended three sampled trees; Kalbi et al. (2017) used four sampled trees for the H-D model developed for Oriental beech stand in Iran. In this study, the selection of 1, 2 and sampled trees had RMSE of 1.199, 1.229 and 1.227, respectively in *P. pinaster*; and 2.122, 2.468 and 2.273 in *P. radiata*. The inclusion of more trees would require additional inventory cost. Thus, the generalized H-D model developed in this study would be valuable to the forest owners as only one sample tree is needed to obtain information of the height of trees in the pine stands.

**TABLE 9** Comparison of fit indices calibration options of the generalized mixed-effect H-D model (equation 37).

| Species | *Calibration options* | *RMSE* | $\bar{R}^2$ | *AIC* | *BIC* |
|---|---|---|---|---|---|
| *P. pinaster* | ONLS | 1.282 | 0.884 | 59908 | 59954 |
| | Mixed model | 1.156 | 0.906 | 56171 | 56218 |
| | One sample tree | 1.199 | 0.899 | 57506 | 57553 |
| | Two sample trees | 1.229 | 0.894 | 58380 | 58426 |
| | Three sample trees | 1.227 | 0.894 | 58326 | 58373 |
| *P. radiata* | ONLS | 2.452 | 0.607 | 35843 | 35871 |
| | Mixed | 1.690 | 0.814 | 30081 | 30108 |
| | One sample tree | 2.122 | 0.706 | 33601 | 33629 |
| | Two sample trees | 2.468 | 0.602 | 35940 | 35968 |
| | Three sample trees | 2.273 | 0.663 | 34669 | 34696 |

## CONCLUSION

This study has introduced new single-variable and a generalized height-diameter model with mixed-effect for *P. pinaster* and *P. radiata*. Fixed-parameters were expanded with random-parameters in a non-linear manner using dominant height, quadratic mean diameter and number of trees per ha as the stand-specific covariates. Its selection is justified for these species because it is the only model of those analyzed that is exempt from the presence of collinearity between its variables, together with the fact that the estimate of the height corresponds to the dominant height when the normal diameter of the tree It is equal to the dominant diameter of the mass. The calibrated response allows accurate results to be obtained with a very small sampling effort, making this approach highly effective and useful in traditional Spanish forest inventories and dynamic forest growth models. Thus, with one sample tree it will be possible to determine the height of all trees in the *P. pinaster* and *P. radiata* stands.

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## Conflict of Interest:

The authors declare that they have no conflict of interest

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